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Time in a quantum mechanical world

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Abstract. In quantum mechanics it is usual to represent physical reality as a vector in Hilbert space at a particular time, with evolution being governed by Schrödinger's equation which involves an externally imposed time parameter. This leads to difficulties if one wishes to regard the universe as a quantum mechanical system, because there should no time external to such a system. The approach in this paper is to represent the totality of reality as a vector in Hilbert space. We show how time evolution follows, where time now is defined in terms of the states of a quantum mechanical clock which is part of the system. Rather than the correlation between the clock states and the states of the rest of the system arising because both are governed by an imposed law involving an external time parameter, it is seen that this correlation is of the separation-independent Einstein–Podolsky–Rosen type. The total reality vector, which incorporates the whole history of the system, is shown to be a zero-energy eigenstate of the system Hamiltonian. We discuss systems of finite and infinite lifetime, and are able to answer the question: what was the state before the initial state? We conclude that the quantum mechanical system of this paper is a reasonable representation of the observed universe.

1. Introduction

In the canonical quantization approach to gravity (see, for example, Dirac 1958, 1959, Komar 1967, Isham and MacCallum 1975, Alvarez 1989 and references therein) some well known difficulties arise. One of these is that the invariance of the ten classical field equations under arbitrary curvilinear coordinate transformations implies that four of these equations are constraints and the Hamiltonian is effectively zero. The usual quantization procedure yields the quantum mechanical result that the system is in a zero energy eigenstate of the Hamiltonian operator. Application of the Schrödinger time-evolution equation involving the corresponding Hamiltonian operator then leads to a 'frozen' dynamics for which nothing seems to happen. As a possible resolution of this difficulty, it has been considered for some time (Alvarez 1989, Unruh and Wald 1989) that the problem may arise from the imposition on the system of an external time parameter, with the Schrödinger equation being applied as a law of evolution in terms of this external time. While this procedure may be appropriate for a system within the universe, it is doubtful whether it can be applied to the universe itself. If one is dealing with such a system it is reasonable to assume that the Hamiltonian must include all observers and the standard clock which defines time measurement (Page and Wootters 1983, Alvarez 1989, Unruh and Wald 1989). Consequently any reasonable approach should ideally *already incorporate* Schrödinger's equation or its equivalent, that is, there should be no need to postulate the form of any time evolution operator, even that applying to the clock. Another difficulty in matching general relativity to quantum mechanics is that physical reality in usual quantum mechanical theory, that

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is, that which is represented by a ray in Hilbert space, refers only to a particular instant of time. In general relativity, time is simply a label attached to a spacelike hypersurface and only histories have physical meaning (see, for example, Unruh and Wald 1989).

In this paper we approach the problem entirely from a quantum mechanical point of view. We begin with the proposition that the *totality* of physical reality can be represented by a single vector in Hilbert space. From this we derive a quantum mechanical model which incorporates time evolution and which appears to be applicable to the world we observe.

2. Expansion of the history vector

Without defining precisely what we mean by the totality of physical reality, we denote it by S and let it be described by a vector in some Hilbert space Ψ. We let the dimensionality of this Hilbert space be $s+1$, remembering that for S to pertain to the observed universe, s should be extremely large or infinite. For the present, however, we keep s arbitrary to maintain generality and consider the limit as $s\to\infty$ later.

There are s mutually orthogonal vectors which are orthogonal to the vector describing S. We label this latter vector as $|E_0\rangle$ and the complete set of vectors as $|E_n\rangle$ with $n=0,1,\ldots,s$. From these vectors we can construct a Hermitian operator

$$\hat{H}=\sum_{n=0}^{s} E_n|E_n\rangle\langle E_n| \tag{2.1}$$

which has eigenvalues E_n, which at present are totally arbitrary. We assign the value $E_0=0$, which gives us

$$\hat{H}|E_0\rangle=0. \tag{2.2}$$

We do not assign particular values to the remaining E_n but restrict them to satisfy

$$E_n=p_n\delta E \tag{2.3}$$

where p_n are arbitrary non-zero integers and δE is a small common factor. From above $p_0=0$. It is clear that E_n with $n\neq 0$ can be made, to within an error of δE, as close to any real value we choose by a suitable choice of p_n.

The vector describing S can be expanded as

$$|E_0\rangle=a_0^{-1}\sum_{n=0}^{r} a_n|E_n\rangle\delta_{n0} \tag{2.4}$$

where $r\leqslant s$. The expansion of the Kronecker delta

$$\delta_{n0}=(s+1)^{-1}\sum_{m=0}^{s}\exp[-\mathrm{i}mp_n2\pi/(s+1)] \qquad \text{for } p_n\leqslant s \tag{2.5}$$

which can be checked by summing the geometric progression, yields

$$|E_0\rangle=[a_0(s+1)]^{-1}\sum_{m=0}^{s}|\phi_m\rangle \tag{2.6}$$

where

$$|\phi_m\rangle=\sum_{n=0}^{r} a_n\exp[-\mathrm{i}mp_n2\pi/(s+1)]|E_n\rangle=\exp[-\mathrm{i}\hat{H}m\delta E^{-1}2\pi/(s+1)]|\phi_0\rangle \tag{2.7}$$

with

$$|\phi_0\rangle = \sum_{n=0}^{r} a_n |E_n\rangle. \tag{2.8}$$

If we specify that the sum of the squares of the moduli of a_n is unity, then all the $|\phi_m\rangle$ are normalized.

We see, therefore, that the vector $|E_0\rangle$ describing S is proportional to

$$|\phi_0\rangle + |\phi_1\rangle + \ldots + |\phi_m\rangle + \ldots + |\phi_s\rangle \tag{2.9}$$

where each $|\phi_m\rangle$ is a normalized superposition of state vectors $|E_n\rangle$ and which can be obtained from $|\phi_{m-1}\rangle$ by the action of the unitary operator $\exp[-\mathrm{i}H\delta E^{-1}2\pi/(s+1)]$. For reasons which will become apparent later we refer to the superposition (2.9) as a 'history' vector or history state.

3. Time

Let us now suppose that S is such that each of the states $|\phi_M\rangle, |\phi_{M+1}\rangle, \ldots, |\phi_{M+P}\rangle$ in a continuous section of the superposition (2.9) can be factorized into the product of two states which are superpositions of basis states of two subspaces Ψ_B and Ψ_C whose tensor product is the Hilbert space Ψ, where Ψ_B has dimensions $b+1$ and Ψ_C has dimensions $c+1$, with $(s+1)=(b+1)(c+1)$. That is, we can write

$$|\phi_m\rangle = |B_m\rangle|C_m\rangle \qquad \text{for } M \leqslant m \leqslant M+P. \tag{3.1}$$

The fact that $|B_m\rangle$ and $|C_m\rangle$ are completely in Ψ_B and Ψ_C respectively for all the values of m shown in (3.1) allows us to factorize the unitary operator transforming $|\phi_m\rangle$ to another state in this section of the history state into two unitary operators involving only the basis states of Ψ_B and Ψ_C respectively. Explicitly we can define

$$\hat{H}_b = \sum_{i=0}^{b} E_{bi}|E_{bi}\rangle\langle E_{bi}| \tag{3.2}$$

$$\hat{H}_c = \sum_{j=0}^{c} E_{cj}|E_{cj}\rangle\langle E_{cj}| \tag{3.3}$$

where the $(s+1)$ states $|E_{bi}\rangle|E_{cj}\rangle$ form the states $|E_n\rangle$. Then

$$\hat{H}|\phi_m\rangle = (\hat{H}_b + \hat{H}_c)|\phi_m\rangle \qquad \text{for } M \leqslant m \leqslant M+P. \tag{3.4}$$

Consistently with this and with (2.7), we can write

$$|B_{m+1}\rangle = \exp[-\mathrm{i}\hat{H}_b\delta E^{-1}2\pi/(s+1)]|B_m\rangle \tag{3.5}$$

$$|C_{m+1}\rangle = \exp[-\mathrm{i}\hat{H}_c\delta E^{-1}2\pi/(s+1)]|C_m\rangle \tag{3.6}$$

for the range of m in (3.4).

We call the factors in (3.1) the states of subsystems B and C respectively. We now particularize to a specific quantum system C by letting the eigenvalues of $\hat{H}_c$ in (3.3) take equally spaced values

$$E_{cj} = j\Delta E \tag{3.7}$$

where ΔE is constant integer multiple of δE. Further, we let the state $|C_M\rangle$ be an equal superposition of the basis states of Ψ_C:

$$|C_M\rangle = (c+1)^{-1} \sum_{j=0}^{c} |E_{cj}\rangle. \tag{3.8}$$

There are $c+1$ such states which are mutually orthogonal:

$$\begin{aligned}|C_{M+k}\rangle &= (c+1)^{-1} \sum_{j=0}^{c} \exp[-\mathrm{i}kj2\pi/(c+1)]|E_{cj}\rangle \\ &= \exp[-\mathrm{i}k\hat{H}_c\Delta E^{-1}2\pi/(c+1)]|C_M\rangle \end{aligned} \tag{3.9}$$

where k takes the values $0, 1, \ldots, c$. The orthogonality of these states, which we call phase states, can be checked directly (compare with the orthonormal set of oscillator phase states discussed by Pegg and Barnett 1988). The next state, that is with $k = c+1$, is identical to $|C_M\rangle$ and so on. We define a special case, which we call an ideal system C, by setting

$$\Delta E = (b+1)\delta E. \tag{3.10}$$

The maximum eigenvalue of $\hat{H}_c$ for this system is $c\Delta E$. The condition $p_n \leqslant s$ in (2.5) restricts the maximum eigenvalue sum E_n to be no greater than $s\delta E$, from which we find that the maximum possible eigenvalue of system B is ΔE. This ideal case is thus quite extreme.

Substituting ΔE from (3.10) into (3.9) and comparing with (3.6), we find that $|C_{M+k}\rangle$ for $k \leqslant P$ is both one of the $c+1$ orthogonal phase states and is also a factor of a term in the history state. Thus the relevant section of the history state (2.9) can be written as

$$|B_M\rangle|C_M\rangle + |B_{M+1}\rangle|C_{M+1}\rangle + \ldots + |B_{M+q}\rangle|C_{M+q}\rangle + \ldots + |B_{M+P}\rangle|C_{M+P}\rangle. \tag{3.11}$$

We note that the state $|C_{M+c+1}\rangle$ is identical with the state $|C_M\rangle$, corresponding to one complete period, so it is convenient to set $P \leqslant c$ to ensure that all phase states of the ideal system C is this history section are orthogonal.

The state (3.11) exhibits an Einstein–Podolsky–Rosen (EPR) type of correlation. In the EPR paradox, two spatially separated systems A and D are in a correlated state $|A_1\rangle|D_1\rangle + |A_2\rangle|D_2\rangle$. Provided $|A_1\rangle$ and $|A_2\rangle$ are orthogonal, then if system A is found in $|A_1\rangle$, the system D must be found in $|D_1\rangle$. In (3.11) the orthogonality of the states $|C_m\rangle$ ensures that there is a distinct phase state for each value of m, that is, there is no overlap between phase states in different terms. We note that the states $|B_m\rangle$ in (3.11) need *not* be orthogonal. Thus, if we have two different states of B, these must be associated with two different phase states, but two different phase states can be associated with the same state of B. For example if $\langle B_m|B_{m+1}\rangle = 1$ then both $|C_m\rangle$ and $|C_{m+1}\rangle$ are associated with $|B_m\rangle$. Thus the system C states, by virtue of their orthogonality, 'set the conditions' for the state of the rest of the system and thus the value of m associated with $|C_m\rangle$ should, in accord with the argument of Unruh and Wald (1989), make a suitable choice for a label to serve as a time value indicator. We therefore define a time interval as being proportional to a difference in m value, that is

$$\Delta t = \delta t \Delta m \tag{3.12}$$

where δt is a constant determined by the units chosen. Then for example, the time difference between the states $|C_M\rangle$ and $|C_{M+q}\rangle$ is $q\delta t$. Because the time differences are defined in terms of phase states of the ideal system C, we call this system an *ideal clock*. Because each state $|B_m\rangle$ in (3.11) is associated with one of the orthogonal clock states, we can also attach to each state the time value of the associated clock state.

Having defined time differences, for states of both systems C and B, it is now reasonable to refer to the generators of the shifts in the time label as Hamiltonians for these systems. From (3.5) and (3.6), it is clear that, at least to within the same constant factor, $\hat{H}_b$ and $\hat{H}_c$ are the Hamiltonians for the system B and the clock respectively. Of course this only holds for the interval, which we can now regard as a time interval, specified by (3.1) during which the clock C and the system B, which we might interpret as the rest of the universe, exist as separately identifiable systems whose states obey (3.1). From (3.4), we see that during this interval these two systems are non-interacting, that is we can write the sum of the two individual Hamiltonians as the total Hamiltonian $\hat{H}$. The correlation between states of B and C at different times arises not through interaction but is an EPR type correlation. A further factorization of the states $|B_m\rangle$ similar to (3.1) for some period enables the introduction of an 'observer' as part of the overall quantum system. The change in state of this observer with m value, that is with time, will also correlate with the change in clock states.

We have mentioned that with a change in the value of m of $c+1$, the state of the clock becomes again identical with its original state. This means that the period of the clock is $(c+1)\delta t$. Equating this to $2\pi/\omega$ where ω is the angular frequency, allows us to write the exponent in (3.9) as

$$-\mathrm{i}k\hat{H}_c\Delta E^{-1}2\pi/(c+1) = -\mathrm{i}\hbar^{-1}\hat{H}_ck\delta t \tag{3.13}$$

where $\hbar$ is defined as the ratio of ΔE, which we can now call the gap between successive energy states of the clock, to the angular frequency.

Further, from (3.10), (2.7) can then be written as

$$|\phi_m\rangle = \exp[-\mathrm{i}\hbar^{-1}\hat{H}m\delta t]|\phi_0\rangle. \tag{3.14}$$

We can, of course, choose our zero of time to correspond to any state and still be consistent with (3.12). For convenience we define $t = m\delta t$, giving

$$|\phi_m\rangle = \exp[-\mathrm{i}\hbar^{-1}\hat{H}t]|\phi_0\rangle. \tag{3.15}$$

The total lifetime T_L which can be ascribed to the total system, that is the time period between $|\phi_0\rangle$ and $|\phi_s\rangle$ is equal to

$$s\delta t = s2\pi\hbar/[\Delta E(c+1)]$$

that is

$$T_L\delta E = s2\pi\hbar/(s+1) \tag{3.16}$$

where we have used, from (3.10),

$$(c+1)\Delta E = (s+1)\delta E. \tag{3.17}$$

Having defined time, we now see why we call $|E_0\rangle$, which describes the totality of reality S and which has the form (2.9), a history vector. This vector, which we now know to be an eigenstate of the Hamiltonian $\hat{H}$ with zero eigenvalue, does not represent the state of the universe at a particular time, instead it is the superposition of all such states at all possible distinct times of an ideal clock.

4. Time evolution equation

Writing $|\phi_{m+1}\rangle - |\phi_m\rangle$ as $\delta|\phi_m\rangle$, we obtain from (3.15)

$$\delta|\phi_m\rangle = [\exp(-\mathrm{i}\hbar^{-1}\hat{H}\delta t) - 1]|\phi_m\rangle. \tag{4.1}$$

If the energy eigenvalues E_n associated with the states $|E_n\rangle$ with significantly non-zero coefficients in the expansion of $|\phi_m\rangle$ are such that $E_n \ll \hbar/\delta t$, we can obtain from (4.1)

$$\delta|\phi_m\rangle/\delta t = -\mathrm{i}\hbar^{-1}\hat{H}|\phi_m\rangle \tag{4.2}$$

with the obvious parallel to the Schrödinger equation. Corresponding expressions would apply for $\delta|C_m\rangle$ and $\delta|B_m\rangle$ in terms of $\hat{H}_C$ and $\hat{H}_B$ under the same restriction. This restriction, however, is quite important. For example, for the ideal clock, the component states of $|C_m\rangle$ are equally weighted and the highest state has, from (3.7), energy E_{cc} such that

$$E_{cc}\delta t = c2\pi\hbar/(c+1), \tag{4.3}$$

where we have used the result, which follows from (3.13), that

$$\Delta E\delta t = 2\pi\hbar/(c+1). \tag{4.4}$$

Thus the Schrödinger equation similar to (4.2) does not apply to the ideal clock itself. The Schrödinger equation may be applicable, however, for isolated subsystems with energies substantially less than $s\delta E$, which from (2.3) and the restriction in (2.5) is the maximum allowed energy $E_{\max}$ of the whole system, and which obeys the relation

$$E_{\max}\delta t = s2\pi\hbar/(s+1) \tag{4.5}$$

which follows from (3.10) and (4.4). The Schrödinger differential equation can only be obtained, of course, if we can let δt tend to zero. Conditions under which this is possible are discussed below.

5. Other clocks

The resolution time δt of the ideal clock, that is the time between successive orthogonal states, is achieved by choosing an impractically high value for ΔE of $(b+1)\delta E$. As discussed earlier this choice restricts the maximum possible energy of system B to be equal to the lowest non-zero energy eigenvalue of the clock. If we wish to use system B to represent the rest of the universe, a more practical clock would have $\Delta E = (b+1)\delta E/K$ where K has a value greater than unity such that $(b+1)/K$ is an integer in accord with (2.3), which means that K is rational. The frequency ω of this clock is correspondingly reduced by a factor K, and the resolution time is $K\delta t$. To study the action of a practical clock, let us choose K to be an integer greater than unity. We then find in the section of the history state corresponding to (3.11) that neighbouring clock states are not orthogonal. However, $|C_M\rangle$ is orthogonal to $|C_{M+K}\rangle$ and $|C_{M+2K}\rangle$ and so on. A clock reading corresponding to a state $|C_{M+K}\rangle$, for example, can be obtained not only if the clock is in this state, but also if it is in any of a number of neighbouring states with a significant overlap with this state. We can show that such states are confined to nearby states $|C_m\rangle$ with m varying in an approximate range of width K centred on M. It follows that the state of B is not precisely determined, but has a high likelihood of being one of the states between $|B_{M+K/2}\rangle$ and $|B_{M+3K/2}\rangle$. Any physical system with equally-spaced energy levels can constitute a clock of the type discussed here. For example as the number of levels increases, the clock states approach the phase states of the harmonic oscillator (Pegg and Barnett 1988), which has long formed the basis for practical clocks. For a finite number of levels, the system also corresponds to a quantum clock similar to that described by Peres (1980).

6. Limiting cases

For $|\phi_m\rangle$ to represent the state of the universe at a particular time, the dimensionality $s+1$ of the Hilbert space Ψ must be extremely large. Whether or not it is infinity is, of course, impossible to determine observationally. Correspondingly we would expect δE and δt to be very small. One other parameter of interest is the lifetime T_{L} of the universe. All of these are related by the expressions we have derived, and we consider various possibilities below.

We consider first the case where T_{L} tends to infinity, corresponding to a universe with an infinite lifetime. From (3.16) this implies immediately that $\delta E \to 0$ and thus, because the maximum allowed energy of the system is $s\delta E$, this means that s must approach infinity as well, in order to allow non-zero energy eigenstates. The result $\delta E \to 0$ means that the difference between possible energy configurations of subsystems and that obtainable by having a completely free choice of energy eigenvalues tends to zero. Thus the universe can contain subsystems representing physical systems such as hydrogen atoms for example, with energy levels precisely in accord with present theoretical models. The effect on δt of letting T_{L} tend to infinity is indeterminate because $\delta t = T_{\mathrm{L}}/s$, so we cannot deduce the size of the minimum time step. However (4.5) shows that it must be very small for any reasonable cosmological model. Thus in a universe with an infinite lifetime, the dimensionality of the Hilbert space is infinite, our condition (2.3) does not restrict possible energy level configurations and the minimum time step of an ideal clock is $2\pi\hbar/E_{\max}$.

The other possibility is that T_{L} is finite. From (3.16) it follows that δE must be non-zero, but there is no restriction on s. This means that there is now a finite difference between the energy eigenvalues of our approach and those obtainable by an entirely free choice. From conventional quantum mechanics, however, we know that an experiment to measure this difference would occupy a time period of at least $2\pi\hbar/\delta E$, which from (3.16) is the lifetime of the universe, so there is no observational reason against the applicability of our approach to a universe with a finite lifetime. There are two possibilities for s with T_{L} finite: (a) s tends to infinity or (b) s is finite. From $\delta t = T_{\mathrm{L}}/s$, we see that in case ($a$) δt must approach zero and thus, for example, we can recover Schrödinger's time-dependent differential equation from (4.2) for a finite energy subsystem. In this case there is a countable infinity of component states with different time labels in (2.8) making up the whole history state of the universe. For case (b) δt is non-zero, there are a finite number of component states with different time labels and we have at best a difference, rather than a differential, equation to describe the time evolution.

7. Discussion

In this paper we have examined the properties of a vector $|E_0\rangle$ which satisfies (2.2) where $\hat{H}$ has now been shown to be the Hamiltonian operator. We have found that this vector is a superposition of vectors each of which can represent, to within an error which is in principle unobservable, states of our observed universe at particular times. For subsystems which have an energy much less than the maximum allowed energy of the universe, the variation of these states from time to time can be described by Schrödinger's equation. For the whole universe the time development is still given by the simple unitary operator $\exp(-i\hbar^{-1}\hat{H}\delta t)$, which transforms $|\phi_m\rangle$ to $|\phi_{m+1}\rangle$.

The expansion

$$|E_0\rangle = \ldots + |\phi_{m-1}\rangle + |\phi_m\rangle + |\phi_{m+1}\rangle + \ldots \tag{7.1}$$

has a simple explanation. If (2.2) holds, that is, if $|E_0\rangle$ is a zero energy eigenstate of $\hat{H}$, then

$$\exp(-i\hbar^{-1}\hat{H}\delta t)|E_0\rangle = |E_0\rangle \tag{7.2}$$

and $|E_0\rangle$ is also an eigenstate of the time displacement operator. Applying the time displacement operator in (7.2) to (7.1) means, ignoring the end states for the moment, changing $|\phi_{m-1}\rangle$ to $|\phi_m\rangle$ and $|\phi_m\rangle$ to $|\phi_{m+1}\rangle$ and so on, which clearly leaves (7.1) unaltered. This will be precisely true if the history state (7.1) extends from $-\infty$ to $+\infty$. The interesting question which arises is for a universe with a finite or semi-infinite lifetime. To examine this, we first choose an arrow of time by saying that the state

$$|\phi_{m+1}\rangle = \exp(-i\hbar^{-1}\hat{H}\delta t)|\phi_m\rangle \tag{7.3}$$

is the state 'after' $|\phi_m\rangle$. The arrow thus points in the direction of increasing m in (2.9). To obtain the state before $|\phi_m\rangle$, we apply the inverse operator. There is now meaning to the question: what is the state *before* $|\phi_0\rangle$? It would be tempting, if one were to revert to some external time reference, to say that, because there is no state before $|\phi_0\rangle$ in (2.9), the action of $\exp(-i\hbar^{-1}\hat{H}\delta t)$ on $|\phi_m\rangle$ must be zero, in a way analogous to that in which the boson annihilation operator destroys the vacuum state thus preventing bosons being found in negative energy states. This cannot apply here, however, because from the definition (2.1), $\hat{H}$ is Hermitian and thus the time shift operator and its inverse are definitely unitary. Indeed, the action on $|\phi_0\rangle$ can be calculated directly. From (2.8)

$$\exp(i\hbar^{-1}\hat{H}\delta t)|\phi_0\rangle = \sum_{n=0}^{r} a_n \exp(i\hbar^{-1}E_n\delta t)|E_n\rangle. \tag{7.4}$$

Substituting from (2.3), and using (3.16) with $\delta t = T_L/s$ allows us to write the exponent in (7.4) as

$$i\hbar^{-1}E_n\delta t = ip_n 2\pi/(s+1) = ip_n 2\pi - ip_n 2\pi s/(s+1). \tag{7.5}$$

Thus, because p_n is an integer,

$$\exp(i\hbar^{-1}E_n\delta t) = \exp[-ip_n 2\pi s/(s+1)] = \exp(-i\hbar^{-1}E_n s\delta t). \tag{7.6}$$

Substituting into (7.4) then yields

$$\exp(i\hbar^{-1}\hat{H}\delta t)|\phi_0\rangle = \exp(-i\hbar^{-1}\hat{H}s\delta t)|\phi_0\rangle = |\phi_s\rangle \tag{7.7}$$

which is just the *final* state of the system. Similarly the state just after the final state $|\phi_s\rangle$ is the initial state $|\phi_0\rangle$. This result, though perhaps surprising, is not unreasonable. The expression $\hat{H}|E_0\rangle = 0$ with $\hat{H}$ Hermitian requires that $|E_0\rangle$ must be an eigenstate, with eigenvalue unity, of the time displacement operator. If the series (7.1) is finite, the state $|E_0\rangle$ is indeed invariant under a shift of $|\phi_m\rangle$ to $|\phi_{m+1}\rangle$ for $m \neq s$ and a shift of $|\phi_s\rangle$ to $|\phi_0\rangle$. Indeed this would seem to be an unavoidable consequence. Of course, by letting the lifetime T_L be long enough, we can postpone the return to the initial state indefinitely. We might comment that this mathematical periodicity for a finite lifetime system does not necessarily imply a reversal of the arrow of time in the later part of the life of the system, that is, the next to last states $|\phi_{s-1}\rangle, |\phi_{s-2}\rangle, \ldots$, are not necessarily the same as the first states $|\phi_0\rangle, |\phi_1\rangle, \ldots$. Nor does the apparent jump from the final to the initial state represent a gross physical discontinuity. These states are connected by a simple unitary transformation and the apparent jump is of the type which occurs when the minute hand of a clock 'jumps back' to zero instead of reading 60.

8. Conclusion

This paper is not an attempt to quantize the classical theory of general relativity; instead it is a purely quantum mechanical approach based on representing, by a vector in an abstract Hilbert space, the totality of reality rather than reality at a particular time. While it does make contact with other approaches through (2.2), that is not its main purpose. The universe is essentially quantum mechanical in nature, and classical theories should ideally arise as limiting cases of quantum mechanics. The reasonably simple approach of this paper provides answers to some questions. For example, the correlated behaviour between a clock and another physical system, which occurs irrespective of their spatial separation, is usually considered to arise because both are governed by a law involving some external time parameter. In our approach this correlation is seen to be simply of the EPR type which, as is well known, is also separation independent. This correlation is a natural consequence of the whole history of the universe being expressible as a single vector which is a superposition of a large number of states with different time labels. Further, the way in which these component states vary with the time label is a *consequence* of the theory, rather than an additional postulate. That is, the single history vector already contains the seeds of Schrödinger's equation. Time has been defined in terms of the states of a particular type of quantum clock, of which the harmonic oscillator is a particular example. If a different type of clock structure were used the clock states may not have the same type of spread over history as those of the clock we have chosen, that is, such a clock might not run uniformly relative to our clock and the Schrödinger equation would have a different form.

While we conclude that the quantum mechanical system discussed in this paper can represent the observed universe, there are of course also questions left unanswered. These include the role of the observer and the unidirectionality of the arrow of time, for example, why an observer's amount of memory changes monotonically with a monotonic change in time values. Our approach does not seem to suggest any underlying reason for this. Concerning the question of the collapse of the wavefunction associated with an act of observation, our approach does not accommodate this concept in the literal sense. Any observer is clearly part of the overall quantum system, and the changes of state of the total system comprising the observer, the system observed and the clock have a well defined time evolution which does seem to accommodate discontinuous changes of state of any one component, unless, perhaps, such a change is somehow balanced by a sudden change in another component of the system, or there is a sudden redefinition of what constitutes each component. The simplest postulate consistent with out approach is that the collapse simply does not occur. We do not explore this any further here.

Finally we should remark that, while some of the ideas incorporated in and arising out of this paper have been considered in other contexts and from different viewpoints by a number of other authors, what is presented here is a reasonably straightforward mathematical formalism which unifies such concepts as consequences of the very simple notion that the totality of reality can be represented as a vector in Hilbert space. There is no need to introduce an external time parameter, indeed, time is part of the reality represented by this vector.

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